Soliton bistability in triply doped fiber with saturating nonlinearity

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Bistable, in the sense of Kaplan [Phys. Rev. Lett. **55** 1291 (1985)], soliton solutions are obtained and studied in the averaged model of the triply doped fiber with saturating nonlinearity. [S1063-651X(97)10010-1]

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INTRODUCTION

Recently, Enns and Edmundson [1] obtained and studied the properties of bistable (in the sense of Kaplan [2]) solitons in a triply doped fiber with saturating nonlinearity. In their work the saturation coefficients of the dopants are related among themselves in a stringent manner in order to fulfill the necessary properties (put forth by Enns, Rangnekar, and Kaplan [3]) of the nonlinear function $f(I)$, I being the light intensity, in the evolution equation. In addition to that, the pulse evolution equation used by them is an unaveraged one, which is not desirable since the transverse distribution of the modal field in the fiber is not uniform over the entire cross section. Because averaging changes the form of the nonlinear function $f(I)$ [4,5] and according to Kaplan [2] the existence of bistable solitons crucially depends on the derivative of $f(I)$ it becomes necessary to study the averaged model.

Consider a monomode fiber with circular cross section. Let z be the direction along the fiber. The nonlinear wave equation in the core region of the fiber can be written as

$$\vec{\nabla}^2\vec{E}-\frac{1}{c^2}\frac{\partial^2\vec{D}^L}{\partial t^2}=\frac{1}{c^2}\frac{\partial^2\vec{D}^{NL}}{\partial t^2}, \qquad (1)$$

where $\vec{D}^L$ and $\vec{D}^{NL}$ are the linear and the nonlinear parts, respectively, of the electric induction vector $\vec{D}$. $\vec{D}^L$ is given by

$$\vec{D}^L=\int_0^\infty \varepsilon(t')\vec{E}(t-t')dt', \qquad (2)$$

while

$$\vec{D}^{NL}=\varepsilon_{NL}(|\vec{E}|^2)\vec{E}, \qquad (3)$$

where ε is the linear permittivity and ε_{NL} is the intensity-dependent nonlinear permittivity. As in the case of a single dopant, for the given case of a triply doped fiber, we take the nonlinear permittivity in the form

$$\varepsilon_{NL}(|\vec{E}|^2)=\sum_{j=1}^{3}\frac{\varepsilon_2^{(j)}|\vec{E}|^2}{1+|\vec{E}|^2/I_s^{(j)}}, \qquad (4)$$

where $\varepsilon_2^{(j)}$ ($j=1,2,3$) are the Kerr coefficients for the nonlinear permittivity of the dopants related to the respective Kerr coefficients for the nonlinear refractive index change of the dopants by [5] $\varepsilon_2^{(j)}=2n_0n_2^{(j)}$, n_0 being the linear refractive index of the fiber core. Here $I_s^{(j)}$ ($j=1,2,3$) are the saturation intensities of the dopants. Note that the intensities $I_s^{(j)}$ have to be given in $\mathrm{V}^2/\mathrm{m}^2$, with the corresponding units for n_2 and ε_2 while calculating the physical quantities related to the soliton pulse.

As usual we represent the electric-field envelope amplitude in the form

$$\vec{E}(x,y,z,t)=\vec{e}R(\vec{r})A(z,t)\exp[-i(\omega_0t-\beta_0z)], \qquad (5)$$

where $\vec{e}$ is the unit vector in the direction of polarization, ω_0 is the carrier frequency, β_0 is the propagation constant, $R(\vec{r})$ is the mode function giving the transverse distribution of the field in the mode, and $A(z,t)$ is the slowly varying complex envelope amplitude of the pulse. In order to derive the differential equation governing pulse propagation in the medium described by Eqs. (2)–(4) we adopt the customary slowly varying envelope approximation (SVEA). Further, assuming the temporal dispersion of the dielectric permittivity to be small, we expand the electric field $\vec{E}(t-t')$ in Eq. (2) into a Taylor series in t' [6] and use the resulting expression to obtain the required series for $\vec{D}^L(t)$. We then differentiate this new expression of $\vec{D}^L(t)$ twice with respect to t to obtain $\partial^2\vec{D}^L/\partial t^2$. We also differentiate $\vec{D}^{NL}(t)$ twice to obtain $\partial^2\vec{D}^{NL}/\partial t^2$. Now substituting for $\vec{E}$ [taking into account Eq. (5)], $\partial^2\vec{D}^L/\partial t^2$, and $\partial^2\vec{D}^{NL}/\partial t^2$ in Eq. (1), using the condition of the SVEA [6], and averaging the resulting equation over the fiber cross section we arrive at the dimensionless evolution equation for the normalized complex envelope amplitude $q(\xi,\tau)$,

$$iq_\xi+\frac{1}{2}q_{\tau\tau}+f(|q|^2)q=0, \qquad (6)$$

where

$$f(|q|^2)=1-\frac{\ln(1+|q|^2)}{|q|^2}+\frac{\mu}{\gamma_1}\left(1-\frac{\ln(1+\gamma_1|q|^2)}{\gamma_1|q|^2}\right)-\frac{\epsilon}{\gamma_2}\left(1-\frac{\ln(1+\gamma_2|q|^2)}{\gamma_2|q|^2}\right). \tag{7}$$

The dimensionless variables in Eqs. (6) and (7) are given by

$$q=\frac{A}{\sqrt{I_s^{(1)}}},\quad \xi=\frac{\omega n_2^{(1)}I_s^{(1)}}{c}z,$$

$$\tau=\sqrt{\frac{\omega n_2^{(1)}I_s^{(1)}}{c(-k_{\omega\omega})}}\left(t-\frac{z}{v_g}\right) \tag{8}$$

and

$$\mu=\frac{n_2^{(2)}}{n_2^{(1)}},\quad \gamma_1=\frac{I_s^{(1)}}{I_s^{(2)}},\quad \epsilon=\frac{|n_2^{(3)}|}{n_2^{(1)}},\quad \gamma_2=\frac{I_s^{(1)}}{I_s^{(3)}}. \tag{9}$$

The averaging is done by taking the first moment of the differential equation, obtained after the above-mentioned substitutions, with respect to the transverse field distribution $R(\vec{r})$, which, since the soliton is supported by the LP_{01} mode, is taken to be Gaussian [7]. For details see Ref. [6].

Note that our investigation has shown that soliton solutions in the given model exist only if one of the dopants is defocusing. Hence we have taken the third dopant with negative Kerr coefficient $n_2^{(3)}$, which has resulted in the negative sign before ϵ in Eq. (7).

SOLITON SOLUTION

We look for the fundamental soliton solutions of Eq. (6) in the form

$$q(\xi,\tau)=\sqrt{\Psi(\tau)}\exp(i\beta\xi), \tag{10}$$

satisfying the boundary conditions

$$\lim_{|\tau|\to\infty}\Psi(\tau)=\lim_{|\tau|\to\infty}[\partial\Psi(\tau)/\partial\tau]=0. \tag{11}$$

The parameter β has the meaning of a nonlinear propagation constant shift. Using this form of the soliton solution we obtain from Eqs. (6), (7), and (10) the ordinary differential equation for $\Psi(\tau)$,

$$\frac{\Psi''}{4\Psi}-\frac{(\Psi')^2}{8\Psi^2}+(1-\beta)+\left(\frac{\mu}{\gamma_1}-\frac{\epsilon}{\gamma_2}\right)-\frac{\ln(1+\Psi)}{\Psi}-\mu\frac{\ln(1+\gamma_1\Psi)}{\gamma_1^2\Psi}+\epsilon\frac{\ln(1+\gamma_2\Psi)}{\gamma_2^2\Psi}=0. \tag{12}$$

Here the prime stands for the ordinary derivative with respect to τ.

In order to obtain the soliton solution for the given initial conditions $\Psi_0\equiv\Psi(\tau=0)$ and $d\Psi/d\tau=0$ at $\tau=0$ we have to integrate Eq. (12) for a given set of parameters γ_1, μ, γ_2, and ϵ. The nonlinear addition to the propagation constant β, required for the integration of Eq. (12), is obtained by taking into account the first integral of Eq. (12) and the boundary conditions (11):

$$\beta=1+\frac{\mu}{\gamma_1}-\frac{\epsilon}{\gamma_2}-\frac{1}{\Psi_0}\left[F_1(\Psi_0)+\frac{\mu}{\gamma_1^2}F_2(\Psi_0)-\frac{\epsilon}{\gamma_2^2}F_3(\Psi_0)\right], \tag{13}$$

where

$$F_1(\Psi_0)=\int_0^{\Psi_0}\frac{\ln(1+\xi)}{\xi}d\xi,$$

$$F_2(\Psi_0)=\int_0^{\Psi_0}\frac{\ln(1+\gamma_1\xi)}{\xi}d\xi,$$

$$F_3(\Psi_0)=\int_0^{\Psi_0}\frac{\ln(1+\gamma_2\xi)}{\xi}d\xi. \tag{14}$$

NUMERICAL RESULTS AND DISCUSSION

For a given set of parameters μ, γ_1, ϵ, and γ_2 and a given initial amplitude $\sqrt{\Psi_0}$ we first determine the soliton shape by numerically integrating Eq. (12) and then calculate the power P of the soliton solution according to the formula [2]

$$P=\int_0^{\Psi_0}\frac{dI}{\sqrt{2[\beta-F(I)]}}, \tag{15}$$

where

$$F(I)=\frac{1}{I}\int_0^I f(I)dI. \tag{16}$$

The results are shown in Fig. 1, where we have plotted P as a function of β for a certain choice of parameters. It is clear from Fig. 1 that $P(\beta)$ becomes N shaped, i.e., multivalued (for a range of ϵ when μ, γ_1, and γ_2 are fixed) and for a given value of P there are three values of β for which soliton solutions exist. We also notice in Fig. 1 that if for a fixed set (μ,γ_1,γ_2) we increase the value of ϵ, the power $P(\beta)$ is initially a single-valued function of β (curve a with $\epsilon=1.75$ and curve b with $\epsilon=1.8$) showing no bistability, but then becomes N shaped (curve c with $\epsilon=1.805$) with the onset of bistability. A further increase in ϵ results in the appearance

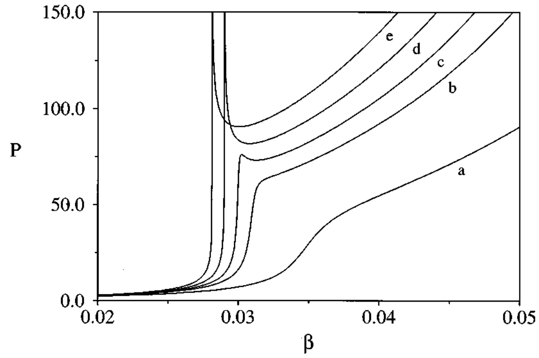

FIG. 1. Soliton power P vs nonlinear propagation constant β for $\gamma_1=5$, $\gamma_2=1.5$, $\mu=1.5$, and $\epsilon=1.78$ (curve a), $\epsilon=1.8$ (curve b), $\epsilon=1.805$ (curve c), $\epsilon=1.81$ (curve d), and $\epsilon=1.815$ (curve e).

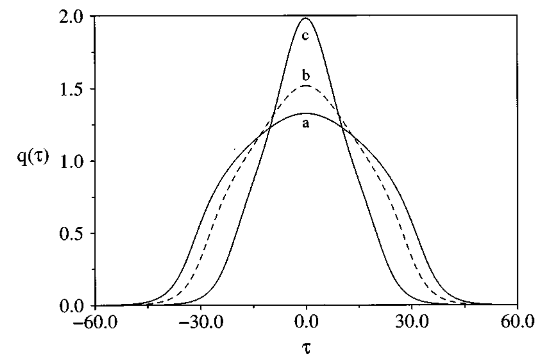

FIG. 2. Soliton solution $\Psi(\tau)$ for $\gamma_1=5$, $\gamma_2=1.5$, $\mu=1.5$, and $\epsilon=1.805$ corresponding to curve c of Fig. 1. Solution a ($\beta=0.030\,124$) and solution c ($\beta=0.032\,537$) belong to the lower and upper positive slope branches, respectively, of $P(\beta)$. Solution b ($\beta=0.030\,471\,5$) belongs to the negative slope branch of $P(\beta)$ and is unstable.

of a pole in $P(\beta)$ at certain β values, which depend on the parameters (curve d with $\epsilon=1.81$ and curve e with $\epsilon=1.82$).

The pole is associated with an interval of Ψ_0 for which no soliton solution exists due to the defocusing action of the third dopant. For example, in Fig. 1 this gap of missing soliton solutions exists for curves d and e and, in fact, this gap exists for all values of the parameter ϵ beyond a critical value, say, ϵ_c. From an experimental point of view, fibers with material properties corresponding to curve c have quite different characteristics from fibers corresponding to curve d or e. In the first case, bistability is encountered only in a finite interval of soliton power, in spite of the fact that soliton solutions exist for all value of peak intensity and soliton pulse widths are bounded. In the second case, i.e., for curves d and e, with ϵ above the critical value ϵ_c, bistable solitons exist only for powers above a certain threshold and the lower and upper stable branches are separated by an interval of peak intensities, for which no soliton solutions can be obtained. Also, the soliton pulse width increases sharply when we approach the pole from below along the lower stable soliton branch. Note that this situation corresponds to the phenomenon of so-called discontinuous solitons predicted recently by Snyder *et al.* [7].

The soliton solutions corresponding to the positive slope branch of the $P(\beta)$ curve satisfy the stability criterion [2] given by $dP/d\beta>0$ and are stable, while those corresponding to the negative slope branch are unstable. For illustration we have depicted the soliton shapes in Fig. 2 corresponding to all three branches of the $P(\beta)$ curve for $\gamma_1=5$, $\gamma_2=1.5$, $\mu=1.5$, and $\epsilon=1.805$. All these solutions have the same power equal to 75. The solutions of Fig. 2(a), corresponding to $\beta=0.030\,123\,4$, and Fig. 2(c), with $\beta=0.032\,53$, are stable, while the solution of Fig. 2(b), corresponding to $\beta=0.030\,471\,5$, is unstable. Inserting typical figures for the fiber parameters [6,8,9] [$\lambda=1.55\,\mu$m, $n_0=1.44$, $n_2^{(1)}=2\times10^{-13}$ cm^2/W, $I_s^{(1)}=200$ MW/cm^2, and $k_{\omega\omega}=-27$ (ps)^2/km] into the rescaling equation (8), we find that one unit of the variable τ corresponds to a time interval of approximately 13 ps. Thus, in the given example the upper branch soliton has a (full width at half maximum) of 215 fs and a peak intensity of $I_{max}=q_{max}^2 I_s^{(1)}=789$ MW/cm^2, while the lower branch soliton has a width of 568 fs and a peak intensity of 352 MW/cm^2. These values lie well within the experimentally accessible and technologically useful range.

We have studied soliton solutions of Eq. (6) for a wide range of the parameters. Our analysis shows that in the given averaged model bistable solitons exist for those values of the parameters for which the nonlinear function $f(I)$ is N-shaped. The corresponding integrated function $F(I)$ can be either monotonic or N-shaped. This behavior is different from the case of the unaveraged model of Ref. [1], where a sharp increase in $f(I)$ is responsible for the existence of bistable solitons. We would like to note here that the averaged model as considered here does not display bistability corresponding to the case of a sharp increase in the nonlinear function f. This might be related to the fact mentioned earlier that averaging changes the form of the nonlinear function whose derivative is crucial for the existence of bistability. An extensive search scanning through a large range of the parameters μ, γ_1, and γ_2 including the case of $f'(0)=0$ and $f''(0)=0$ (similar to Ref. [1]) could not yield a positive result.

CONCLUSION

We have obtained and studied bistable solitons in the averaged model of triply doped fibers with nonlinear saturation in the refractive index of the core. The soliton solutions have the same power, but different shapes corresponding to different values of the nonlinear propagation constant β as predicted by Kaplan [2]. Two stable branches of solitons are separated by an unstable branch as required for switching from one bistable state to the other [10]. We have shown that bistable solitons can exist without requiring the specific behavior of a sharp increase in the nonlinear function $f(I)$. Also we have shown the existence of the so-called discontinuous solitons in a realistic model.

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